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parts together ; and from which a formula for a greater number of squares might be suggested. Such a principle is found to govern the generation of the four-square results, when these are arrived at by a peculiar process, which the author exhibits. The same process is then extended to the case of eight squares ; and it is found that

$$\begin{aligned}
 & (s'^2 + t'^2 + u'^2 + v'^2 + w'^2 + x'^2 + y'^2 + z'^2) \times \\
 & (s^2 + t^2 + u^2 + v^2 + w^2 + x^2 + y^2 + z^2) = \\
 & (ss' + tt' + uu' + vv' + ww' + xx' + yy' + zz')^2 \\
 & + (st' - ts' + uv' - vu' + ux' - xw' + yz' - zy')^2 \\
 & + (su' - us' + vt' - tv' + yw' - wy' + xz' - zx')^2 \\
 & + (sv' - vs' + tu' - ut' + wz' - zw' + xy' - yx')^2 \\
 & + (sw' - ws' + xt' - tx' + uy' - yu' + zv' - vz')^2 \\
 & + (sx' - xs' + tw' - wt' + vy' - vy' + zu' - uz')^2 \\
 & + (sy' - ys' + zt' - tz' + vx' - xv' + wu' - uw')^2 \\
 & + (sz' - zs' + ty' - yt' + vw' - wv' + ux' - xu')^2.
 \end{aligned}$$

These results are verified by the actual development of the several squares ; which development, by the mutual cancelling of all the double products, reduces itself to the sixty-four squares furnished by the product of the proposed factors, when multiplied together in the ordinary way.

The author then enters into a more minute examination of the constitution of the preceding polynomial ; and shows that the cancelling of the aforesaid double products is a necessary consequence of that constitution.

It is further shown that the product continues to be of the same form as each of the factors, when the coefficients $a^0, a^1, a^2, a^3, \&c.$, are introduced in order, in connexion with the squares entering those factors.

Sir William Rowan Hamilton stated also a theorem respecting products of sums of eight squares, which does not essentially differ from the foregoing, and was communicated to him by John T. Graves, Esq., about the end of the year 1843.

One form of the theorem is the following:

$$\begin{aligned}
 & (00' - 11' - 22' - 33' - 44' - 55' - 66' - 77')^2 \\
 & + (10' + 01' - 32' + 23' - 54' + 45' - 76' + 67')^2 \\
 & + (20' + 31' + 02' - 13' - 74' - 65' + 56' + 47')^2 \\
 & + (30' - 21' + 12' + 03' - 64' + 75' + 46' - 57')^2 \\
 & + (40' + 51' + 72' + 63' + 04' - 15' - 36' - 27')^2 \\
 & + (50' - 41' + 62' - 73' + 14' + 05' - 26' + 37')^2 \\
 & + (60' + 71' - 52' - 43' + 34' + 25' + 06' - 17')^2 \\
 & + (70' - 61' - 42' + 53' + 24' - 35' + 16' + 07')^2 \\
 = & (0^2 + 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2) (0'^2 + \\
 & 1'^2 + 2'^2 + 3'^2 + 4'^2 + 5'^2 + 6'^2 + 7'^2).
 \end{aligned}$$

In a letter dated January 18th, 1844, Mr. Graves communicated to Sir William Hamilton his theorem respecting sums of eight squares under the form :

$$\begin{aligned}
 & (a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2) \times \\
 & (a'^2 + b'^2 + c'^2 + d'^2 + e'^2 + f'^2 + g'^2 + h'^2) = \\
 & a'^2 + b'^2 + c'^2 + d'^2 + e'^2 + f'^2 + g'^2 + h'^2;
 \end{aligned}$$

where $a'' \dots h''$ denoted the following expressions:

$$\begin{aligned}
 a'' &= aa' - bb' - cc' - dd' - ee' - ff' - gg' - hh'; \\
 b'' &= ba' + ab' - dc' + cd' - fe' + ef' + hg' - gh'; \\
 c'' &= ca' + db' + ac' - bd' - ge' - hf' + eg' + fh'; \\
 d'' &= da' - cb' + bc' + ad' - he' + gf' - fg' + eh'; \\
 e'' &= ea' + fb' + gc' + hd' + ae' - bf' - cg' - dh'; \\
 f'' &= fa' - eb' + hc' - gd' + be' + af' + dg' - ch'; \\
 g'' &= ga' - hb' - ec' + fd' + ce' - df' + ag' + bh'; \\
 h'' &= ha' + gb' - fc' - ed' + de' + cf' - bg' + ah'.
 \end{aligned}$$

In a letter of somewhat earlier date, but evidently written in haste, upon a journey, and dated December 26th, 1843, analogous expressions had been given, containing, however, some errors in the signs, which were soon afterwards corrected as above. That earlier letter also indicated an expectation that a theory of *octaves*, including a new and extended system of imaginaries, which had thus been suggested to the writer (J. T. Graves, Esq.) by Sir William R. Hamilton's theory of

quaternions, might itself be extended so as to form a theory of what Mr. Graves at the time proposed to call 2^m -*ions*: but in a letter written shortly afterwards, doubts were expressed respecting the possibility of this additional extension, from octaves to sets of sixteen.

The special thanks of the Academy were given to Lord Farnham, for his Donation to the Museum of the following Collection of Antiquities:

Two Gun Barrels, one Gun Lock, three Cannon Balls, several fragments of Cannon Balls and Shells, four Leaden Bullets, and one Sword, from Cloughoughter Castle.

Curious ancient Keys, from Newtownbarry, and Bally-jamesduff.

Fifty-five Flint Arrow-Heads.

Six Flint Knives.

Twenty-four Bronze Celts, of different patterns.

An ancient Hand-mill, perfect; the top Stone of another; and a round Grind Stone.

An old Glass Bottle, from Cavan.

A Sling Stone, from Dunshaughlin.

One perfect Bronze Sword, found at Corchor, Co. Cavan, and fragments of another.

Five Blades of Bronze Daggers.

Two Bronze Spears.

Four Bronze Pins, with Rings.

Two Bronze Bosses.

A Brass Spoon.

A Brass Spur, found at Shaneloon, Co. Cavan.

A Brass Cross.

A Silver Cross.

A Brass Pistol.

A Snuff Box made from a Cannon used at the Siege of Derry.

A Snuff Box found on the battle ground at Vinegar Hill.

A fragment of an ancient illuminated Breviary, found in a bog at Myshal, Co. Carlow.

Two large Brass Pots, and an Iron one, from the Lake of Virginia.

An ancient Bronze Bucket-shaped Pot.

Three Meters.

A small Wooden Cup.

Four Grey-beard Jars.

One beautiful cinerary Urn, found at Killinagh, “ lying in the centre of six large stones, placed perpendicularly, and opposite to each other, three at either side. Near the spot was a large, rough flag, supposed to have been originally covering the stones. The urn was buried about two feet under ground, and when found was nearly full of ashes, and the place about bore evident marks of fire ; no lid was found, nor any other marks of its being a place of burial.”—Extracted from the Rev. C. S. Montgomery’s Note to Lord Farnham.

Two Shoes found in a bog.

Ten Spindle Stones, and two Weights.

Two Stone Hammers.

One square perforated Stone.

One triangular Jet Ornament.

Two Flint and eighteen Stone Celts.

One Sharpening Stone.

A Stone shaped like a Cow’s Foot.

Two perfect Bronze Trumpets, and one imperfect, found at Coracanway, Co. Cavan.

Three Iron Stirrups.

A fragment of shed Horn of the Irish Elk, found at Lisduff, Co. Cavan.

A Letter, found in the pocket of the Rev. Mr. Murphy, who was killed at Arklow, on the 9th of June, 1798.